routine work, but uniformity and an initial, precise fit of the piston-cylinder system are still of prime importance for the establishment of a primary scale. The fundamental advantage of a controlled-clearance gage is the elimination, in principle at least, of the distortion of the cylinder. In any of the theoretical approaches to the distortion of the piston-cylinder system, the change in effective area due to the distortion of the cylinder is several times the change due to the distortion of the piston, and the two effects add. The size of the area correction term for the controlled clearance gage is thus reduced appreciably and changes its sign. Furthermore, the analysis of the cylinder using elasticity theory is more complicated and less reliable than that for the piston.

In the operation of the controlled-clearance gage, the cylinder is forced by means of the jacket pressure to fit the piston. How well this fit can be made is a limiting factor in the accuracy of the technique. For routine operation of moderate accuracy and precision, a measurement of the torque required to rotate the piston will tell when the two are well matched but not closed. Extreme care must be taken to avoid mechanical contact and friction. Operation of the gage is significantly more elaborate, as discussed below, when the highest precision is desired and if excessive wear and scoring of the assembly are to be prevented. Since the deformation due to the jacket pressure P_j is assumed linear, the jacket pressure P_j' necessary to completely close the crevice is given in terms of the measured pressure P_m by

$$P_j' = P_{j0}' + LP_m \tag{6}$$

where P_{j0}' is the pressure required to close the crevice at atmospheric pressure. Values of L, according to Johnson and Newhall, vary between 0.35 and 0.7 for usable gages. The value of P_{j0}', of course, is highly dependent on the initial clearance. The values of P_{j0}' and L must be experimentally obtained as discussed below.

In a controlled-clearance gage as designed by Johnson and Newhall, the piston is made to extend from the cylinder at both ends by more than one piston diameter. This arrangement tends to eliminate nonuniform deformation of the piston due to end effects and makes more reasonable the use of simple elastic theory. Johnson, et al. (1957) presented a rather general mathematical development of the change in effective area of a piston constrained as is the case in a controlled-clearance piston gage. They assumed a radially isotropic piston material, perfectly cylindrical geometry, and uniform end-loading of the piston. Using an analysis based on the integration of the deformation caused by a series of differential pressure steps along the piston, they showed that the effective area is equal to the average of the area at the top and the bottom of the piston provided the two ends are far removed (more than one diameter) from any pressure gradient. No assumption was necessary concerning the shape or pressure profile along the length of the piston. For isotropic piston materials this conclusion leads to an expression for λ of

$$\lambda = \frac{(3\sigma - 1)}{E} \tag{7}$$

assuming the piston fits the cylinder perfectly. This equation yields an approximate value of -8×10^{-8}/bar for a carboloy piston. For highly accurate pressure determinations, the gage is balanced for one given P_m at several different settings of P_j, and the leakage of liquid through the crevice is measured at each jacket pressure. This fluid-flow rate can be easily measured by noting the fall rate of the piston itself provided the rest of the pressure chamber is completely free of leaks. Since the deformation of the cylinder with jacket pressure should be linear, and since the viscous-flow rate between two parallel surfaces varies as the cube of the separation of the surfaces, a plot of the jacket pressure against the cube root of the flow rate should yield a straight line provided the crevice is uniform. The linearity of such a plot gives an excellent check of how well the piston and cylinder fit. A nonlinear plot immediately illustrates scoring, noncylindrical geometry, or a leaky chamber. If a gage is used to measure very high pressures (greater than ten kbar), some nonlinearity may be introduced by the break down of linear elastic theory. An extrapolation of this linear plot to zero flow-rate is now taken as the required jacket pressure for a perfect fit of the cylinder to the piston. Corrections must then be made of the measured value P_m at a very low leak to an idealized value of no leak when the cylinder would perfectly fit the piston.

The use of this technique and extrapolation procedure is referred to as "measurement with no leak" although all measurements are actually made with measurable leakage past the piston. The correction from the low-leak condition to the "no leak" condition is of the order of 0.1 percent at ten kbar. Newhall, et al. (1963), used simple elastic theory to make this correction. Bennett and Vodar (1963) have analyzed with greater care the leakage flow and have illustrated an alternative but more complex method of making this final correction to the zero-leak condition based upon more general elastic theory and additional experimental viscosity data. The analysis of Bennett and Vodar indicates that the pressure gradient is highest near the upper region of the cylinder, but their work confirms the simple analysis of the change in effective area with pressure given above. Cross (1964) assumes the area of the cylinder A_c can be written

$$A_c = A_0\,[1 - b(P_j' - P_j)] \tag{8}$$

and determines the constant b by measuring the change in chamber pressure P_m with an auxiliary gage of very high sensitivity as the jacket pressure P_j is changed. Since P_j' is obtained from the extrapolation mentioned above, A_c is experimentally determined. It is interesting to note that in the limit of zero fluid flow in the "no-leak" condition, the fluid velocity itself is zero, and frictional forces in the crevice need not be considered. The effective area can then be determined from geometrical conditions only.

The action of the viscous fluid in the crevice as it narrows to a few microns average thickness is still an unknown feature of the gage. When the crevice approaches such a small average thickness, there exists obvious variation in this thickness both around the perimeter and along the length. For example, from data taken at a low-leak condition, Bennett and Vodar (1963) predicted jacket pressure necessary to "seize" the piston. They then proceeded to decrease the crevice dimension by applying jacket pressure until mechanical and/or electrical contact was made. In all cases, the measured contact was experienced at a lower jacket pressure than predicted. This obviously indicates the existence of high spots and argues strongly in favor of the extrapolation procedure over a direct measure of contact. Yet, one still wonders whether the extrapolated fit is consistent with the techniques used in measuring the piston area at atmospheric conditions or whether the atmospheric measurement was also of high points. It is rather evident that the use of a larger-diameter piston-cylinder assembly will reduce the percentage uncertainty associated with the crevice flow near zero leak. This fact is one reason for the increased reliability of large-diameter piston systems used at the lower pressure. At the higher pressures, however, the handling of the excessively large weights required to load the larger pistons creates other rather severe problems.

Yasunami (1967a, 1967b) has reported the construction of a controlled-clearance gage with a diameter of 1.1 centimeters usable to 10 kbar in which he has used a lever to multiply the gravitational force associated with the weights rather than applying the weights directly to the piston. Adequate details have not been reported in the open literature to allow an evaluation of systematic errors involved in this work. Since errors in lever arm as well as undetermined frictional forces in the pivot point cause first-order errors in the effective area of the gage, precise evaluation of possible errors is of fundamental importance. The problem is intensified by the large force (approximately ten tons in Yasunami's work) which the pivot must support. Zhokhovskii, et al. (1959) and Konyaev (1961) have used an auxiliary free-piston "hydraulic-multiplier" system in a similar manner to apply higher loads to a regular free-piston gage. Some approach such as this appears necessary if one is to increase the accuracy and reliability of the controlled-clearance piston gage at very high pressures.

The temperature of the piston under operating conditions may be considerably higher than the rest of the apparatus due to the viscous friction within the crevice. Newhall, et al. (1963) crudely estimated at 20 °C increase above ambient temperature and treated it as if the total piston were at this temperature. The 20 °C estimate is much larger than is generally expected for the effect. Since uncertainties of the order of 0.01 percent result from errors of 10 °C, attention must be paid to this item if greater accuracy is desired.

The major development and use of controlled-clearance gages have been for pressures below 10 kbar although a commercial unit rated at 14 kbar maximum pressure is available. Johnson and Heydemann (1967) and Heydemann (1967) have recently developed a controlled-clearance gage usable to 26 kbar. They incorporated Bridgman's tapered support-ring principle to decrease the distortion of the cylinder and also included a jacket-pressure chamber to obtain fine adjustment of the crevice. In their particular design it was not possible to supply additional fluid to the chamber without disturbing the crevice clearance and thus they were unable to evaluate the constant b in equation (8). Due to the complex cylinder support they were unable to use elastic theory to extrapolate to the "no-leak" condition, and, as a result, they obtained much lower accuracy (60 bar in 25 kbar) than that for which one would hope. Nevertheless, they did demonstrate the feasibility of making measurements at 26 kbar, and it is apparent that with appropriate changes in design such a gage can be built and operated using proper extrapolation procedures. When one reaches pressures of this magnitude, nonlinear terms in the elastic coefficients of the piston become significant and must be considered. Such effects give uncertainties of approximately 0.01 percent at 10 kbar.

c. Similarity Method

Dadson and coworkers (1955, 1958, 1965) have developed two methods for evaluating the elastic distortion in regular free-piston gages based upon an underlying assumption that the functional variation of the gap width along the length of the crevice will be closely related for two different free-piston gages, even though neither is known. They assume this relationship between the two systems can be related to dimensions and elastic parameters for known systems of specific construction as discussed below. Dadson names these two methods the "similarity" method and the "flow" method. He has carried out extensive work using the former method and has used the flow method only as a supporting measurement.

The similarity method assumes that two idealized piston-cylinder systems constructed of isotropic elastic material, having perfect cylindrical and concentric geometry, and with selected known elastic moduli can be constructed such that the functional variation of the gap width along the length of the crevice is proportional at all pressures. The proportionality constant in this case is the inverse ratio of the elastic moduli of the